# 第3章

# 拓扑变换下多智能体切换系统的分布式优化控制

## 3.1 问题描述

### 3.1.1 系统描述

考虑一类高阶非线性严格反馈多智能体切换系统如下：

$$\begin{cases} \dot{x}_{i,1}(t) = x_{i,2} + g_{i,1}^{\chi_i(t)}(x_{i,1}) \\ \dot{x}_{i,l}(t) = x_{i,l+1} + g_{i,l}^{\chi_i(t)}(x_{i,1}, x_{i,2}, \cdots, x_{i,l}) \\ \dot{x}_{i,n}(t) = u_i(t) + g_{i,n}^{\chi_i(t)}(x_{i,1}, x_{i,2}, \cdots, x_{i,n}) \\ y_i(t) = x_{i,1}(t) \end{cases}, \quad i = 1, 2, \cdots, N, l = 2, 3, \cdots, n-1 \tag{3-1}$$

式中：$u_i(t)$为控制输入；$y_i(t)$为系统输出，$g_{i,l}^{\chi_i(t)}(x_{i,l}, x_{i,2}, \cdots, x_{i,l})$为定义在系统状态上的未知非线性函数，$\chi(t)$为分段连续函数，用来描述子系统之间切换的触发条件，当$\chi(t) = q$时，意味着第$q$个子系统处在活动状态。

定义第 $i$ 个智能体的系统状态向量 $\boldsymbol{X}_{i,l}=(x_{i,1},x_{i,2},\cdots,x_{i,l})^{\mathrm{T}}\in\mathbb{R}^{l}$。将系统式(3-1)改写如下：

$$\begin{cases}\dot{\boldsymbol{X}}_{i,n}=\boldsymbol{A}_i\boldsymbol{X}_{i,n}+\boldsymbol{K}_i y_i+\sum_{l=1}^{n}\boldsymbol{B}_{i,l}[g_{i,l}^{q}(\boldsymbol{X}_{i,l})]+\boldsymbol{B}_i u_i(t)\\ y_i=\boldsymbol{C}_i\boldsymbol{X}_{i,n}\end{cases}\tag{3-2}$$

式中

$$\boldsymbol{A}_i=\begin{bmatrix}-k_{i,l} & & & \\ \vdots & & I_{n-1} & \\ & & & \\ -k_{i,n} & 0 & \cdots & 0\end{bmatrix},\quad \boldsymbol{K}_i=\begin{bmatrix}k_{i,1}\\ \vdots\\ k_{i,n}\end{bmatrix},\quad \boldsymbol{B}_i=\begin{bmatrix}0\\ \vdots\\ 1\end{bmatrix},$$

$$\boldsymbol{B}_{i,l}=[0\cdots1\cdots0]^{\mathrm{T}},\quad \boldsymbol{C}_i=[1\quad 0\cdots0]$$

给定一个正定矩阵 $\boldsymbol{Q}_i^{\mathrm{T}}=\boldsymbol{Q}_i$，存在一个正定矩阵 $\boldsymbol{P}_i^{\mathrm{T}}=\boldsymbol{P}_i$ 并满足

$$\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\boldsymbol{A}_i=-2\boldsymbol{Q}_i\tag{3-3}$$

### 3.1.2 构造含惩罚项的优化问题

考虑路径追踪问题的局部目标函数设计如下：

$$\begin{aligned}f_i(x_{i,1})&=a_i(x_{i,1}-x_{\mathrm{d}})^2+c\\&=a_i x_{i,1}^2+b_i x_{i,1}+c_i\end{aligned}\tag{3-4}$$

式中：$x_{\mathrm{d}}$ 为智能体追踪的目标信号；$a_i>0,b_i=-2a_i x_{\mathrm{d}},c_i=a_i x_{\mathrm{d}}^2+c,1\leqslant i\leqslant N$ 且 $a_i$、$c$ 是常数。定义全局目标函数 $f:\mathbb{R}^N\to\mathbb{R}$ 为

$$f(\boldsymbol{x}_1)=\sum_{i=1}^{N}f_i(x_{i,1})\tag{3-5}$$

考虑局部目标函数 $f_i$ 是可导的强凸函数，全局目标函数 $f$ 也是可导的强凸函数。定义向量 $\boldsymbol{x}_1=[x_{1,1},x_{2,1},\cdots,x_{N,1}]^{\mathrm{T}}$。根据引理 1.3，对于某一常数 $\alpha\in\mathbb{R}$，若有 $\boldsymbol{x}_1=\alpha\cdot\boldsymbol{1}_N$，则可得

$$\boldsymbol{L}^{\kappa(t)}\boldsymbol{x}_1=0\tag{3-6}$$

基于上式，设计如下惩罚项：

$$\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{L}^{\kappa(t)}\boldsymbol{x}_1=0\tag{3-7}$$

定义如下惩罚函数：

$$P(\boldsymbol{x}_1)=\sum_{i=1}^{N} f_i(x_{i,1})+\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{L}^{\kappa(t)}\boldsymbol{x}_1 \tag{3-8}$$

因为全局目标函数是强凸函数，可以得到惩罚函数也是强凸函数的结论。

本章的目标是设计控制器($u_1,\cdots,u_N$)，来对每个 $i=1,\cdots,N$，使得 $\lim\limits_{t\to\infty} x_{i,1}\to x_{i,1}^*$。定义向量 $\boldsymbol{x}_1^*=(x_{1,1}^*,\cdots,x_{N,1}^*)$，其中第 $i$ 个智能体的分布式优化问题最优解 $x_{1,1}^*$ 定义如下：

$$(x_{1,1}^*,\cdots,x_{N,1}^*)=\underset{(x_{1,1},\cdots,x_{N,1})}{\operatorname{argmin}} P(\boldsymbol{x}_1) \tag{3-9}$$

## 3.2　自适应神经网络反演控制器设计

### 3.2.1　神经网络观测器设计

由于系统中的非线性项 $g_{i,l}^q(\boldsymbol{X}_{i,l})$未知，因此有如下假设：

**假设 3.1**　根据 RBF 神经网络逼近技术，假设未知函数 $g_i^q(\boldsymbol{X}_i)$可以表示为

$$g_i^q(\boldsymbol{X}_i \mid \boldsymbol{\theta}_i)=\boldsymbol{\theta}_i^{\mathrm{T}}\boldsymbol{\psi}_i(\boldsymbol{X}_i),\quad 1\leqslant i\leqslant n \tag{3-10}$$

式中：$\boldsymbol{\theta}_i$ 为理想常数向量；$\boldsymbol{\psi}_i(\boldsymbol{X}_i)$为高斯基函数向量。

设计基于 RBF 神经网络的状态观测器为

$$\begin{aligned}\dot{\hat{\boldsymbol{X}}}_{i,n}&=\boldsymbol{A}_i\hat{\boldsymbol{X}}_{i,n}+\boldsymbol{K}_i y_i+\sum_{l=1}^{n}\boldsymbol{B}_{i,l}[\hat{g}_{i,l}^q(\hat{\boldsymbol{X}}_{i,l}\mid\boldsymbol{\theta}_{i,l})]+\boldsymbol{B}_i u_i(t)\\ \hat{y}_i&=\boldsymbol{C}_i\hat{\boldsymbol{X}}_{i,n}\end{aligned} \tag{3-11}$$

式中：$\boldsymbol{C}=[1\cdots0\cdots0]$；$\hat{\boldsymbol{X}}_{i,l}=(\hat{x}_{i,1},\hat{x}_{i,2},\cdots,\hat{x}_{i,n})^{\mathrm{T}}$ 是 $\boldsymbol{X}_{i,l}=(x_{i,1},x_{i,2},\cdots,x_{i,l})^{\mathrm{T}}$ 的估计值。

定义系统的观测误差 $\boldsymbol{e}_i=\boldsymbol{X}_{i,n}-\hat{\boldsymbol{X}}_{i,n}$，根据式(3-2)和式 (3-11)可得

$$\dot{\boldsymbol{e}}_i=\boldsymbol{A}_i\boldsymbol{e}_i+\sum_{l=1}^{n}\boldsymbol{B}_{i,l}[g_{i,l}^q(\hat{\boldsymbol{X}}_{i,l})-\hat{g}_{i,l}^q(\hat{\boldsymbol{X}}_{i,l}\mid\boldsymbol{\theta}_{i,l})+\Delta g_{i,l}^q] \tag{3-12}$$

式中

$$\Delta g_{i,l}^q=g_{i,l}^q(\boldsymbol{X}_{i,l})-g_{i,l}^q(\hat{\boldsymbol{X}}_{i,l})$$

通过 RBF 神经网络逼近技术可得

$$\hat{g}^q_{i,l}(\hat{\boldsymbol{X}}_{i,l} \mid \boldsymbol{\theta}_{i,l}) = \boldsymbol{\theta}^{\mathrm{T}}_{i,l}\boldsymbol{\varphi}_{i,l}(\hat{\boldsymbol{X}}_{i,l}) \tag{3-13}$$

定义最优参数向量为

$$\boldsymbol{\theta}^*_{i,l} = \operatorname{argmin}_{\boldsymbol{\theta}_{i,l} \in \boldsymbol{\Omega}_{i,l}} \left[ \sup_{\hat{\boldsymbol{X}}_{i,l} \in \boldsymbol{U}_{i,l}} \mid \hat{g}^q_{i,l}(\hat{\boldsymbol{X}}_{i,l} \mid \boldsymbol{\theta}_{i,l}) - g^q_{i,l}(\hat{\boldsymbol{X}}_{i,l}) \mid \right], \quad 1 \leqslant l \leqslant n \tag{3-14}$$

式中：$\boldsymbol{\Omega}_i$、$\boldsymbol{U}_i$ 分别为变量 $\boldsymbol{\theta}_{i,l}$、$\hat{\boldsymbol{X}}_{i,l}$ 的紧集。

定义最小逼近误差和参数估计误差为

$$\begin{cases} \varepsilon^q_{i,l} = g^q_{i,l}(\hat{\boldsymbol{X}}_{i,l}) - \hat{g}^q_{i,l}(\hat{\boldsymbol{X}}_{i,l} \mid \boldsymbol{\theta}^*_{i,l}) \\ \tilde{\boldsymbol{\theta}}_{i,l} = \boldsymbol{\theta}^*_{i,l} - \boldsymbol{\theta}_{i,l} \end{cases}, \quad l = 1, 2, \cdots, n \tag{3-15}$$

**假设 3.2** 假设最优逼近误差有界，存在已知正常数 $\varepsilon_0$，使得$|\boldsymbol{\varepsilon}_i| \leqslant \varepsilon_0$。

**假设 3.3** 存在一组已知常数 $\gamma_{i,l}$，使得以下关系成立：

$$\mid g_{i,l}(\boldsymbol{X}_{i,l}) - g_{i,l}(\hat{\boldsymbol{X}}_{i,l}) \mid \leqslant \gamma_{i,l} \| \boldsymbol{X}_{i,l} - \hat{\boldsymbol{X}}_{i,l} \| \tag{3-16}$$

根据式(3-11)和式(3-12)可得

$$\begin{aligned} \dot{\boldsymbol{e}}_i &= \boldsymbol{A}_i \boldsymbol{e}_i + \sum_{l=1}^{n} \boldsymbol{B}_{i,l} [g^q_{i,l}(\hat{\boldsymbol{X}}_{i,l}) - \hat{g}^q_{i,l}(\hat{\boldsymbol{X}}_{i,l} \mid \boldsymbol{\theta}_{i,l}) + \Delta g^q_{i,l}] \\ &= \boldsymbol{A}_i \boldsymbol{e}_i + \sum_{l=1}^{n} \boldsymbol{B}_{i,l} [\varepsilon^q_{i,l} + \Delta g^q_{i,l} + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}_{i,l} \boldsymbol{\varphi}_{i,l}(\hat{\boldsymbol{X}}_{i,l})] \\ &= \boldsymbol{A}_i \boldsymbol{e}_i + \Delta \boldsymbol{g}^q_i + \boldsymbol{\varepsilon}^q_i + \sum_{l=1}^{n} \boldsymbol{B}_{i,l} [\tilde{\boldsymbol{\theta}}^{\mathrm{T}}_{i,l} \boldsymbol{\varphi}_{i,l}(\hat{\boldsymbol{X}}_{i,l})] \end{aligned} \tag{3-17}$$

式中

$$\boldsymbol{\varepsilon}^q_i = [\varepsilon^q_{i,1}, \cdots, \varepsilon^q_{i,n}]^{\mathrm{T}}, \quad \Delta \boldsymbol{g}^q_i = [\Delta g^q_1, \cdots, \Delta g^q_n]^{\mathrm{T}}。$$

选取 Lyapunov 函数：

$$V_0 = \sum_{i=1}^{N} V_{i,0} = \sum_{i=1}^{N} \frac{1}{2} \boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i \boldsymbol{e}_i \tag{3-18}$$

对式(3-18)进行微分可得

$$\begin{aligned} \dot{V}_0 &\leqslant \sum_{i=1}^{N} \left\{ \frac{1}{2} \boldsymbol{e}^{\mathrm{T}}_i (\boldsymbol{P}_i \boldsymbol{A}^{\mathrm{T}}_i + \boldsymbol{A}_i \boldsymbol{P}_i) \boldsymbol{e}_i + \boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i (\boldsymbol{\varepsilon}^q_i + \Delta \boldsymbol{g}^q_i) + \sum_{l=1}^{n} \boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i \boldsymbol{B}_{i,l} [\tilde{\boldsymbol{\theta}}^{\mathrm{T}}_{i,l} \boldsymbol{\varphi}_{i,l}(\hat{\boldsymbol{X}}_{i,l})] \right\} \\ &\leqslant \sum_{i=1}^{N} \left[ -\boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{Q}_i \boldsymbol{e}_i + \boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i (\boldsymbol{\varepsilon}^q_i + \Delta \boldsymbol{g}^q_i) + \boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i \sum_{l=1}^{n} \boldsymbol{B}_{i,l} \tilde{\boldsymbol{\theta}}^{\mathrm{T}}_{i,l} \boldsymbol{\varphi}_{i,l}(\hat{\boldsymbol{X}}_{i,l}) \right] \end{aligned} \tag{3-19}$$

通过 Young's 不等式以及假设 3.3 可得

$$\boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i (\boldsymbol{\varepsilon}^q_i + \Delta \boldsymbol{g}^q_i) \leqslant \mid \boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i \boldsymbol{\varepsilon}^q_i \mid + \mid \boldsymbol{e}^{\mathrm{T}}_i \boldsymbol{P}_i \Delta \boldsymbol{g}^q_i \mid$$

$$
\begin{aligned}
&\leqslant \frac{1}{2}\|\boldsymbol{e}_i\|^2+\frac{1}{2}\|\boldsymbol{P}\boldsymbol{\varepsilon}_i^q\|^2+\frac{1}{2}\|\boldsymbol{e}_i\|^2+\frac{1}{2}\|\boldsymbol{P}_i\|^2\|\Delta\boldsymbol{g}_i^q\|^2\\
&\leqslant \|\boldsymbol{e}_i\|^2+\frac{1}{2}\|\boldsymbol{P}_i\boldsymbol{\varepsilon}_i^q\|^2+\frac{1}{2}\|\boldsymbol{P}_i\|^2\sum_{l=1}^{n}|\Delta g_{i,l}^q|^2\\
&\leqslant \|\boldsymbol{e}_i\|^2+\frac{1}{2}\|\boldsymbol{e}_i\|^2\|\boldsymbol{P}_i\|^2\sum_{l=1}^{n}\gamma_{i,l}^{q2}+\frac{1}{2}\|\boldsymbol{P}_i\boldsymbol{\varepsilon}_i^q\|^2\\
&\leqslant \|\boldsymbol{e}_i\|^2\left(1+\frac{1}{2}\|\boldsymbol{P}_i\|^2\sum_{l=1}^{n}\gamma_{i,l}^{q2}\right)+\frac{1}{2}\|\boldsymbol{P}_i\boldsymbol{\varepsilon}_i^q\|^2
\end{aligned}\tag{3-20}
$$

以及

$$
\begin{aligned}
&\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i\sum_{l=1}^{n}\boldsymbol{B}_{i,l}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\boldsymbol{\varphi}_{i,l}(\hat{\boldsymbol{X}}_{i,l})\\
&\leqslant \frac{1}{2}\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{e}_i+\frac{1}{2}\sum_{l=1}^{n}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\boldsymbol{\varphi}_{i,l}(\hat{\boldsymbol{X}}_{i,l})\boldsymbol{\varphi}_{i,l}^{\mathrm{T}}(\hat{\boldsymbol{X}}_{i,l})\tilde{\boldsymbol{\theta}}_{i,l}\\
&\leqslant \frac{1}{2}\lambda_{i,\max}^2(\boldsymbol{P}_i)\|\boldsymbol{e}_i\|^2+\frac{1}{2}\sum_{l=1}^{n}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}
\end{aligned}\tag{3-21}
$$

式中：$\lambda_{i,\max}(\boldsymbol{P}_i)$为正定矩阵 $\boldsymbol{P}_i$ 的最大特征值。

根据式(3-19)～式(3-21)可得

$$
\dot{V}_0\leqslant\sum_{i=1}^{N}\left(-q_{i,0}\|\boldsymbol{e}_i\|^2+\frac{1}{2}\|\boldsymbol{P}_i\boldsymbol{\varepsilon}_i^q\|^2+\frac{1}{2}\sum_{l=1}^{n}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}\right)\tag{3-22}
$$

式中

$$
0<\boldsymbol{\varphi}_{i,l}(\cdot)\boldsymbol{\varphi}_{i,l}^{\mathrm{T}}(\cdot)\leqslant 1;\ q_{i,0}=\lambda_{i,\min}(\boldsymbol{Q}_i)-\left(1+\frac{1}{2}\|\boldsymbol{P}_i\|^2\sum_{l=1}^{n}\gamma_{i,l}^2+\frac{1}{2}\lambda_{i,\max}^2(\boldsymbol{P}_i)\right)
$$

式(3-22)改写为

$$
\dot{V}_0\leqslant -q_0\|\boldsymbol{e}\|^2+\frac{1}{2}\|\boldsymbol{P}\boldsymbol{\varepsilon}\|^2+\sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}\tag{3-23}
$$

式中：$q_0=\sum_{i=1}^{N}q_{i,0}$。

### 3.2.2　分布式控制器设计

本节结合自适应反演控制、观测器技术和动态面控制(Dynamic Surface Control，DSC)技术设计虚拟控制律、控制输入和参数自适应律。定义误差变量：

$$\begin{cases} s_{i,1}=x_{i,1}-x_{i,1}^{*} \\ s_{i,l}=\hat{x}_{i,l}-v_{i,l}\ , \quad l=2,\cdots,n \\ w_{i,l}=v_{i,l}-x_{i,l}^{*} \end{cases} \tag{3-24}$$

式中：$s_{i,l}$ 为误差面；$v_{i,l}$ 为滤波器输出；$x_{i,l}^{*}$ 为虚拟控制律；$w_i$ 为 $v_{i,l}$ 和 $x_{i,l}^{*}$ 的误差。

第 1 步　计算惩罚函数式(3-8)的梯度值：

$$\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}=\mathrm{vec}\left(\frac{\partial f_i(x_{i,1}(t))}{\partial x_{i,1}}\right)+\boldsymbol{L}^{\kappa(t)}\boldsymbol{x}_1 \tag{3-25}$$

式中：$\mathrm{vec}\left(\frac{\partial f_i(x_{i,1}(t))}{\partial x_{i,1}}\right)$为元素$\frac{\partial f_i(x_{i,1}(t))}{\partial x_{i,1}}$的列向量。

由于惩罚函数 $P(\boldsymbol{x}_1)$是一个强凸函数，那么可以得到分布式优化问题的最优解满足如下形式：

$$\frac{\partial P(\boldsymbol{x}_1^{*})}{\partial \boldsymbol{x}_1^{*}}=0 \tag{3-26}$$

根据式(3-8)和式(3-26)可得

$$\frac{\partial f_i(x_{i,1}^{*}(t))}{\partial x_{i,1}^{*}}+\sum_{j\in N_i}a_{ij}(x_{i,1}^{*}-x_{j,1}^{*})=0 \tag{3-27}$$

由式(3-4)和式(3-27)可得

$$2a_i(x_{i,1}^{*}-x_{\mathrm{d}})+\sum_{j\in N_i}a_{ij}(x_{i,1}^{*}-x_{j,1}^{*})=0 \tag{3-28}$$

根据式(3-24)和式(3-28)可得

$$\begin{aligned}\frac{\partial P(\boldsymbol{x}_1)}{\partial x_{i,1}}&=\frac{\partial f_i(x_{i,1}(t))}{\partial x_{i,1}}+\sum_{j\in N_i}a_{ij}(x_{i,1}-x_{j,1})\\&=2a_i(x_{i,1}-x_{\mathrm{d}})+\sum_{j\in N_i}a_{ij}(x_{i,1}-x_{j,1})\\&=2a_i(x_{i,1}-x_{\mathrm{d}})+\sum_{j\in N_i}a_{ij}(x_{i,1}-x_{j,1})-2a_i(x_{i,1}^{*}-x_{\mathrm{d}})+\sum_{j\in N_i}a_{ij}(x_{i,1}^{*}-x_{j,1}^{*})\\&=2a_is_{i,1}+\sum_{j\in N_i}a_{ij}(s_{i,1}-s_{j,1})\end{aligned} \tag{3-29}$$

取 $\boldsymbol{s}_1=[s_{1,1},\cdots,s_{N,1}]^{\mathrm{T}}$，根据式(3-29)可得

$$\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}=\boldsymbol{H}^{\kappa(t)}\boldsymbol{s}_1 \tag{3-30}$$

式中：$\boldsymbol{H}^{\kappa(t)}=\boldsymbol{A}+\boldsymbol{L}^{\kappa(t)}$；$\boldsymbol{A}=\mathrm{diag}\{2a_i\}$。

构造 Lyapunov 函数：

$$V_1=V_0+\frac{1}{2}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}\right)+\sum_{i=1}^{N}\frac{1}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,1}$$

$$=V_0+\frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{s}_1+\sum_{i=1}^{N}\frac{1}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,1} \tag{3-31}$$

式中：$\sigma_{i,1}$ 为设计参数。

根据式(3-1)、式(3-11)和式(3-24)可得

$$\dot{s}_{i,1}=\hat{x}_{i,2}+\boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1}+\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1}+\Delta g_{i,1}^{q}+\varepsilon_{i,1}^{q}+e_{i,2} \tag{3-32}$$

由式(3-31)和式(3-32)可得

$$\begin{aligned}\dot{V}_1&=\dot{V}_0+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\dot{\boldsymbol{s}}_1+\sum_{i=1}^{N}\frac{1}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,1}\\&=\dot{V}_0+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}(\hat{\boldsymbol{x}}_2+\mathrm{vec}(\boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})+\mathrm{vec}(\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})+\Delta\boldsymbol{g}_1^{q}+\boldsymbol{\varepsilon}_1^{q}+\boldsymbol{e}_2)+\\&\quad\sum_{i=1}^{N}\frac{1}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\theta}}}_{i,1}\\&=\dot{V}_0+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}(\boldsymbol{s}_2+\boldsymbol{w}_2+\boldsymbol{x}_2^{*}+\mathrm{vec}(\boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})+\mathrm{vec}(\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})+\Delta\boldsymbol{g}_1^{q}+\boldsymbol{\varepsilon}_1^{q}+\boldsymbol{e}_2)+\\&\quad\sum_{i=1}^{N}\frac{1}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\theta}}}_{i,1}\\&=\dot{V}_0+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{s}_2+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{w}_2+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}(\boldsymbol{x}_2^{*}+\mathrm{vec}(\boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})+\mathrm{vec}(\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1}))+\\&\quad\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\Delta\boldsymbol{g}_1^{q}+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\varepsilon_1^{q}+\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{e}_2-\sum_{i=1}^{N}\frac{1}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\dot{\boldsymbol{\theta}}_{i,1}\end{aligned} \tag{3-33}$$

式中：$\boldsymbol{s}_2=[s_{1,2},s_{2,2},\cdots,s_{N,2}]^{\mathrm{T}}$；$\boldsymbol{w}_2=[w_{1,2},w_{2,2},\cdots,w_{N,2}]^{\mathrm{T}}$；$\boldsymbol{x}_2^{*}=[x_{1,2}^{*},x_{2,2}^{*},\cdots,x_{N,2}^{*}]^{\mathrm{T}}$；$\Delta\boldsymbol{g}_1^{q}=[\Delta g_{1,1}^{q},\Delta g_{2,1}^{q},\cdots,\Delta g_{N,1}^{q}]^{\mathrm{T}}$；$\boldsymbol{\varepsilon}_1^{q}=[\varepsilon_{1,1}^{q},\varepsilon_{2,1}^{q},\cdots,\varepsilon_{N,1}^{q}]^{\mathrm{T}}$；$\boldsymbol{e}_2=[e_{1,2},e_{2,2},\cdots,e_{N,2}]^{\mathrm{T}}$；$\mathrm{vec}(\boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})$和 $\mathrm{vec}(\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})$为列向量。

根据 Young's 不等式可得

$$\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{s}_2\leqslant\frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1+\frac{1}{2}\boldsymbol{s}_2^{\mathrm{T}}\boldsymbol{s}_2 \tag{3-34}$$

$$\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{w}_2\leqslant\frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1+\frac{1}{2}\boldsymbol{w}_2^{\mathrm{T}}\boldsymbol{w}_2 \tag{3-35}$$

$$\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\Delta\boldsymbol{g}_1^q \leqslant \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{\gamma}_1^q\boldsymbol{\gamma}_1^{q\mathrm{T}}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{e}_1 \tag{3-36}$$

$$\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{\varepsilon}_1^q \leqslant \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{\varepsilon}_1^{q\mathrm{T}}\boldsymbol{\varepsilon}_1^q \tag{3-37}$$

$$\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{e}_2 \leqslant \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_2 \tag{3-38}$$

式中：$\boldsymbol{\gamma}_1^q=\mathrm{diag}[\gamma_{i,1}^q]$；$\boldsymbol{e}_1=[e_{1,1},e_{2,1},\cdots,e_{N,1}]^{\mathrm{T}}$。

将式(3-34)～式(3-38)代入式(3-33)可得

$$\begin{aligned}\dot V_1 \leqslant{}& \dot V_0 + \boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}(\boldsymbol{x}_2^* + \mathrm{vec}(\boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1}) + \mathrm{vec}(\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1})) + \\ & \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{w}_2^{\mathrm{T}}\boldsymbol{w}_2 + \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{s}_2^{\mathrm{T}}\boldsymbol{s}_2 + \\ & \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{\gamma}_1^q\boldsymbol{\gamma}_1^{q\mathrm{T}}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{e}_1 + \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{\varepsilon}_1^{q\mathrm{T}}\boldsymbol{\varepsilon}_1^q + \\ & \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \frac{1}{2}\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_2 - \sum_{i=1}^{N}\frac{1}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\dot{\boldsymbol{\theta}}_{i,1}\end{aligned} \tag{3-39}$$

与第 2 章相同，可得如下等式：

$$\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{s}_1 = \sum_{i=1}^{N}[2a_i(x_{i,1}-x_{\mathrm{d}}) + \sum_{j\in N_i}a_{ij}(x_{i,1}-x_{j,1})]^2 \tag{3-40}$$

$$\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}\boldsymbol{\gamma}_1\boldsymbol{\gamma}_1^{\mathrm{T}}\boldsymbol{H}^{T}\boldsymbol{s}_1 = \sum_{i=1}^{N}\gamma_{i,1}^2\left[2a_i(x_{i,1}-x_{\mathrm{d}}) + \sum_{j\in N_i}a_{ij}(x_{i,1}-x_{j,1})\right]^2 \tag{3-41}$$

根据式(3-39)～式(3-41)，设计第 1 步虚拟控制律 $x_{i,2}^*$ 和自适应律$\boldsymbol{\theta}_{i,1}$：

$$x_{i,2}^* = -c_{i,1}\left[2a_i(x_{i,1}-x_d) + \sum_{j\in N_i}a_{ij}(x_{i,1}-x_{j,1})\right] - \boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\varphi}_{i,1}(\hat{\boldsymbol{X}}_{i,1}) \tag{3-42}$$

$$\dot{\boldsymbol{\theta}}_{i,1} = \sigma_{i,1}\boldsymbol{\varphi}_{i,1}(\hat{\boldsymbol{X}}_{i,1})\left[2a_i(x_{i,1}-x_{\mathrm{d}}) + \sum_{j\in N_i}a_{ij}(x_{i,1}-x_{j,1})\right] - \rho_{i,1}\boldsymbol{\theta}_{i,1} \tag{3-43}$$

式中：$c_{i,1}=\dfrac{5}{2}+\dfrac{\gamma_{i,1}^{q2}}{2}$和 $\rho_{i,1}$ 为设计参数。

将式(3-42)和式(3-43)代入式(3-39)可得

$$\begin{aligned}\dot V_1 \leqslant{}& -q_0\|\boldsymbol{e}\|^2 + \frac{1}{2}\|\boldsymbol{P}\boldsymbol{\varepsilon}\|^2 + \sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l} - \frac{1}{2}\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)}\boldsymbol{H}^{\kappa(t)\mathrm{T}}\boldsymbol{s}_1 + \\ & \frac{1}{2}\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_2 + \frac{1}{2}\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{e}_1 + \frac{1}{2}\boldsymbol{\varepsilon}_1^{q\mathrm{T}}\boldsymbol{\varepsilon}_1^q + \sum_{i=1}^{N}\frac{\rho_{i,1}}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\theta}_{i,1} + \frac{1}{2}\boldsymbol{s}_2^{\mathrm{T}}\boldsymbol{s}_2 + \frac{1}{2}\boldsymbol{w}_2^{\mathrm{T}}\boldsymbol{w}_2\end{aligned} \tag{3-44}$$

式中

$$q_1=q_0-N;\quad \eta_1=\frac{1}{2}\|\boldsymbol{P\varepsilon}\|^2+\frac{1}{2}\boldsymbol{\varepsilon}_1^{q\mathrm{T}}\boldsymbol{\varepsilon}_1^q$$

通过使用滤波器技术可以获得状态变量 $v_{i,2}$ 为

$$\lambda_{i,2}\dot{v}_{i,2}+v_{i,2}=x_{i,2}^*,\quad v_{i,2}(0)=x_{i,2}^*(0) \tag{3-45}$$

进一步,由式(3-24)和式(3-45)可得

$$\dot{w}_{i,2}=\dot{v}_{i,2}-\dot{x}_{i,2}^*=-\frac{v_{i,2}-x_{i,2}^*}{\lambda_{i,2}}-\dot{x}_{i,2}^*=-\frac{w_{i,2}}{\lambda_{i,2}}+B_{i,2} \tag{3-46}$$

式中:$\lambda_{i,2}$ 为设计参数;$B_{i,2}=-\dot{x}_{i,2}^*$,根据相关文献可知,存在一个正整数 $M_{i,2}$,使得 $|B_{i,2}|\leqslant M_{i,2}$。

第 2 步　定义误差变量 $s_{i,2}=\hat{x}_{i,2}-v_{i,2}$,进一步可得

$$\begin{aligned}\dot{s}_{i,2}&=\dot{\hat{x}}_{i,2}-\dot{v}_{i,2}\\&=\hat{x}_{i,3}+k_{i,2}e_{i,1}+\boldsymbol{\theta}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}+\tilde{\boldsymbol{\theta}}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}+\varepsilon_{i,2}^q+\Delta g_{i,2}^q-\dot{v}_{i,2}\\&=s_{i,3}+w_{i,3}+x_{i,3}^*+k_{i,2}e_{i,1}+\boldsymbol{\theta}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}+\tilde{\boldsymbol{\theta}}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}+\varepsilon_{i,2}^q+\Delta g_{i,2}^q-\dot{v}_{i,2}\end{aligned} \tag{3-47}$$

构造 Lyapunov 函数:

$$V_2=V_1+\sum_{i=1}^N V_{i,2}=V_1+\frac{1}{2}\sum_{i=1}^N\left\{s_{i,2}^2+\frac{1}{\sigma_{i,2}}\tilde{\boldsymbol{\theta}}_{i,2}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,2}+w_{i,2}^2\right\} \tag{3-48}$$

根据 Young's 不等式可得

$$s_{i,2}k_{i,2}e_{i,1}\leqslant\frac{1}{2}s_{i,2}^2+\frac{1}{2}k_{i,2}^2\|e_{i,1}\|^2 \tag{3-49}$$

$$s_{i,2}(s_{i,3}+w_{i,3})\leqslant s_{i,2}^2+\frac{1}{2}(s_{i,3}^2+w_{i,3}^2) \tag{3-50}$$

将式(3-49)、式(3-50)代入式(3-48)可得

$$\begin{aligned}\dot{V}_2\leqslant\dot{V}_1+\sum_{i=1}^N\Bigg[&s_{i,2}(x_{i,3}^*+\boldsymbol{\theta}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}+\tilde{\boldsymbol{\theta}}_{i,32}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}-\dot{v}_{i,2})+\frac{5}{2}s_{i,2}^2+\frac{1}{2}(s_{i,3}^2+w_{i,3}^2)+\\&\frac{1}{2}k_{i,2}^2\|e_{i,1}\|^2+\frac{1}{2}\|\varepsilon_{i,2}^q\|^2+\frac{1}{2}\gamma_{i,2}^{q2}\|e_{i,2}\|^2-\frac{1}{\sigma_{i,2}}\tilde{\boldsymbol{\theta}}_{i,2}^{\mathrm{T}}\dot{\boldsymbol{\theta}}_{i,2}+w_{i,2}\dot{w}_{i,2}\Bigg]\end{aligned} \tag{3-51}$$

设计第 2 步虚拟控制律和自适应律如下:

$$x_{i,3}^*=-c_{i,2}s_{i,2}-3s_{i,2}-\boldsymbol{\theta}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}(\hat{\boldsymbol{X}}_{i,2})+\frac{x_{i,2}^*-v_{i,2}}{\lambda_{i,2}} \tag{3-52}$$

$$\dot{\boldsymbol{\theta}}_{i,2}=\boldsymbol{\sigma}_{i,2}\boldsymbol{\varphi}_{i,2}(\hat{\boldsymbol{X}}_{i,2})s_{i,2}-\rho_{i,2}\boldsymbol{\theta}_{i,2} \tag{3-53}$$

式中：$\rho_{i,2}$ 为设计参数。

将式(3-52)、式(3-53)和式(3-44)代入式(3-51)可得

$$\begin{aligned}\dot{V}_2\leqslant&-q_1\|\boldsymbol{e}\|^2+\eta_1+\sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}+\\&\sum_{i=1}^{N}\frac{\rho_{i,1}}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\theta}_{i,1}-\frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)+\\&\sum_{i=1}^{N}\frac{1}{2}s_{i,2}^2+\sum_{i=1}^{N}\frac{1}{2}w_{i,2}^2+\sum_{i=1}^{N}\left[s_{i,2}\left(-c_{i,2}s_{i,2}-3s_{i,2}-\boldsymbol{\theta}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}(\hat{\boldsymbol{X}}_{i,2})+\right.\right.\\&\left.\frac{x_{i,2}^{*}-v_{i,2}}{\lambda_{i,2}}+\boldsymbol{\theta}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}+\tilde{\boldsymbol{\theta}}_{i,2}^{\mathrm{T}}\boldsymbol{\varphi}_{i,2}-\dot{v}_{i,2}\right)+\\&\frac{5}{2}s_{i,2}^2+\frac{1}{2}(s_{i,3}^2+w_{i,3}^2)+\frac{1}{2}k_{i,2}^2\|e_{i,1}\|^2+\frac{1}{2}\|\varepsilon_{i,2}^{q}\|^2+\frac{1}{2}\gamma_{i,2}^{q2}\|e_{i,2}\|^2-\\&\left.\frac{1}{\sigma_{i,2}}\tilde{\boldsymbol{\theta}}_{i,2}^{\mathrm{T}}(\sigma_{i,2}\boldsymbol{\varphi}_{i,2}(\hat{\boldsymbol{X}}_{i,2})s_{i,2}-\rho_{i,2}\boldsymbol{\theta}_{i,2})+w_{i,2}\left(-\frac{w_{i,2}}{\lambda_{i,2}}+B_{i,2}\right)\right]\end{aligned} \tag{3-54}$$

根据 Young's 不等式，$w_{i,2}B_{i,2}\leqslant\frac{1}{2}w_{i,2}^2+\frac{1}{2}M_{i,2}^2$ 成立，由此可得

$$\begin{aligned}\dot{V}_2\leqslant&-q_2\|\boldsymbol{e}\|^2+\eta_2+\sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}-\\&\frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)+\sum_{i=1}^{N}\frac{\rho_{i,1}}{\sigma_{i,1}}\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\theta}_{i,1}+\\&\sum_{i=1}^{N}\frac{\rho_{i,2}}{\sigma_{i,2}}\tilde{\boldsymbol{\theta}}_{i,2}^{\mathrm{T}}\boldsymbol{\theta}_{i,2}-\sum_{i=1}^{N}c_{i,2}s_{i,2}^2-\sum_{i=1}^{N}\left(\frac{1}{\lambda_{i,2}}-1\right)w_{i,2}^2+\\&\sum_{i=1}^{N}\left[\frac{1}{2}M_{i,2}^2+\frac{1}{2}(s_{i,3}^2+w_{i,3}^2)\right]\end{aligned} \tag{3-55}$$

式中

$$q_2=q_1-\frac{1}{2}\sum_{i=1}^{N}(k_{i,2}^2+\gamma_{i,2}^{q2}) \tag{3-56}$$

$$\eta_2=\eta_1+\frac{1}{2}\sum_{i=1}^{N}\|\varepsilon_{i,2}^{q}\|^2 \tag{3-57}$$

使用滤波器技术可以获得状态变量 $v_{i,2}$ 为

$$\lambda_{i,3}\dot{v}_{i,3}+v_{i,3}=x_{i,3}^{*},\quad v_{i,3}(0)=x_{i,3}^{*}(0) \tag{3-58}$$

进一步,由式(3-26)和式(3-58)可得

$$\dot{w}_{i,3}=\dot{v}_{i,3}-\dot{x}_{i,3}^{*}=-\frac{v_{i,3}-x_{i,3}^{*}}{\lambda_{i,3}}-\dot{x}_{i,3}^{*}=-\frac{w_{i,3}}{\lambda_{i,3}}+B_{i,3} \tag{3-59}$$

式中：$\lambda_{i,3}$ 为设计参数；$B_{i,3}=-\dot{x}_{i,3}^{*}$,存在一个正整数 $M_{i,3}$,使得$|B_{i,3}|\leqslant M_{i,3}$。

第 $m$ 步　定义误差变量 $s_{i,m}=\hat{x}_{i,m}-v_{i,m}$,其导数为

$$\begin{aligned}\dot{s}_{i,m}&=\dot{\hat{x}}_{i,m}-\dot{v}_{i,m}\\&=\hat{x}_{i,m+1}+k_{i,m}e_{i,1}+\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\varepsilon_{i,m}^{q}+\Delta g_{i,m}^{q}-\dot{v}_{i,m}\\&=s_{i,m+1}+w_{i,m+1}+x_{i,m+1}^{*}+k_{i,m}e_{i,1}+\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\varepsilon_{i,m}^{q}+\Delta g_{i,m}^{q}-\dot{v}_{i,m}\end{aligned} \tag{3-60}$$

构造 Lyapunov 函数：

$$V_m=V_{m-1}+\sum_{i=1}^{N}V_{i,m}=V_{m-1}+\frac{1}{2}\sum_{i=1}^{N}\left\{s_{i,m}^{2}+\frac{1}{\sigma_{i,m}}\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,m}+w_{i,m}^{2}\right\} \tag{3-61}$$

式中：$\sigma_{i,m}$ 为设计参数。

对 Lyapunov 函数求导可得

$$\dot{V}_m=\dot{V}_{m-1}+\sum_{i=1}^{N}\left\{s_{i,m}\dot{s}_{i,m}+\frac{1}{\sigma_{i,m}}\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\theta}}}_{i,m}+w_{i,m}\dot{w}_{i,m}\right\} \tag{3-62}$$

将式(3-60)代入式(3-62)可得

$$\begin{aligned}\dot{V}_m=\dot{V}_{m-1}+\sum_{i=1}^{N}[&s_{i,m}(s_{i,m+1}+w_{i,m+1}+x_{i,m+1}^{*}+k_{i,m}e_{i,1}+\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\\&\varepsilon_{i,m}^{q}+\Delta g_{i,m}^{q}-\dot{v}_{i,m})+\frac{1}{\sigma_{i,m}}\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\theta}}}_{i,m}+w_{i,m}\dot{w}_{i,m}]\end{aligned} \tag{3-63}$$

根据 Young's 不等式可得

$$s_{i,m}k_{i,m}e_{i,1}\leqslant\frac{1}{2}s_{i,m}^{2}+\frac{1}{2}k_{i,m}^{2}\|e_{i,1}\|^{2} \tag{3-64}$$

$$s_{i,m}(s_{i,m+1}+w_{i,m+1})\leqslant s_{i,m}^{2}+\frac{1}{2}(s_{i,m+1}^{2}+w_{i,m+1}^{2}) \tag{3-65}$$

$$s_{i,m}\varepsilon_{i,m}^{q}\leqslant\frac{1}{2}s_{i,m}^{2}+\frac{1}{2}\|\varepsilon_{i,m}^{q}\|^{2} \tag{3-66}$$

$$s_{i,m}\Delta g_{i,m}^{q}\leqslant\frac{1}{2}s_{i,m}^{2}+\frac{1}{2}\gamma_{i,m}^{q2}\|e_{i,m}\|^{2} \tag{3-67}$$

将式(3-64)～式(3-67)代入式(3-63)可得

$$\dot{V}_m \leqslant \dot{V}_{m-1}+\sum_{i=1}^{N}\left[s_{i,m}(x_{i,m+1}^{*}+\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}-\dot{v}_{i,m})+\frac{5}{2}s_{i,m}^{2}+\frac{1}{2}(s_{i,m+1}^{2}+w_{i,m+1}^{2})+\frac{1}{2}k_{i,m}^{2}\|e_{i,1}\|^{2}+\frac{1}{2}\|\varepsilon_{i,m}^{q}\|^{2}+\frac{1}{2}\gamma_{i,m}^{q2}\|e_{i,m}\|^{2}-\frac{1}{\sigma_{i,m}}\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\dot{\boldsymbol{\theta}}_{i,m}+w_{i,m}\dot{w}_{i,m}\right] \tag{3-68}$$

设计第 $m$ 步虚拟控制律和自适应律如下：

$$x_{i,m+1}^{*}=-c_{i,m}s_{i,m}-3s_{i,m}-\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}(\hat{\boldsymbol{X}}_{i,m})+\frac{x_{i,m}^{*}-v_{i,m}}{\lambda_{i,m}} \tag{3-69}$$

$$\dot{\boldsymbol{\theta}}_{i,m}=\sigma_{i,m}\boldsymbol{\varphi}_{i,m}(\hat{\boldsymbol{X}}_{i,m})s_{i,m}-\rho_{i,m}\boldsymbol{\theta}_{i,m} \tag{3-70}$$

式中：$\rho_{i,m}$ 为设计参数。

使用滤波器技术可以获得状态变量 $v_{i,m}$ 为

$$\lambda_{i,m}\dot{v}_{i,m}+v_{i,m}=x_{i,m}^{*},\quad v_{i,m}(0)=x_{i,m}^{*}(0) \tag{3-71}$$

进一步，由式(3-26)和式(3-71)可得

$$\dot{w}_{i,m}=\dot{v}_{i,m}-\dot{x}_{i,m}^{*}=-\frac{v_{i,m}-x_{i,m}^{*}}{\lambda_{i,m}}-\dot{x}_{i,m}^{*}=-\frac{w_{i,m}}{\lambda_{i,m}}+B_{i,m} \tag{3-72}$$

式中：$\lambda_{i,m}$ 为设计参数；$B_{i,m}=-\dot{x}_{i,m}^{*}$，存在一个正整数 $M_{i,m}$，使得$|B_{i,m}|\leqslant M_{i,m}$。

将式(3-69)、式(3-70)和式(3-72)代入式(3-68)可得

$$\dot{V}_m \leqslant \dot{V}_{m-1}+\sum_{i=1}^{N}\left[s_{i,m}\left(-c_{i,m}s_{i,m}-3s_{i,m}-\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}(\hat{\boldsymbol{X}}_{i,m})+\frac{x_{i,m}^{*}-v_{i,m}}{\lambda_{i,m}}+\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}-\dot{v}_{i,m}\right)+\frac{5}{2}s_{i,m}^{2}+\frac{1}{2}(s_{i,m+1}^{2}+w_{i,m+1}^{2})+\frac{1}{2}k_{i,m}^{2}\|e_{i,1}\|^{2}+\frac{1}{2}\|\varepsilon_{i,m}^{q}\|^{2}+\frac{1}{2}\gamma_{i,m}^{q2}\|e_{i,m}\|^{2}-\frac{1}{\sigma_{i,m}}\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}(\sigma_{i,m}\boldsymbol{\varphi}_{i,m}(\hat{\boldsymbol{X}}_{i,m})s_{i,m}-\rho_{i,m}\boldsymbol{\theta}_{i,m})+w_{i,m}\left(-\frac{w_{i,m}}{\lambda_{i,m}}+B_{i,m}\right)\right] \tag{3-73}$$

根据 Young’s 不等式，$w_{i,m}B_{i,m}\leqslant\frac{1}{2}w_{i,m}^{2}+\frac{1}{2}M_{i,m}^{2}$ 成立，由此可得

$$\dot{V}_m \leqslant \dot{V}_{m-1}+\sum_{i=1}^{N}\left[s_{i,m}\left(-c_{i,m}s_{i,m}-3s_{i,m}-\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\frac{x_{i,m}^{*}-v_{i,m}}{\lambda_{i,m}}+\boldsymbol{\theta}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}+\right.\right.$$

$$\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}\boldsymbol{\varphi}_{i,m}-\dot{v}_{i,m})+\frac{5}{2}s_{i,m}^{2}+\frac{1}{2}(s_{i,m+1}^{2}+w_{i,m+1}^{2})+\frac{1}{2}k_{i,m}^{2}\|e_{i,1}\|^{2}+$$
$$\frac{1}{2}\|\varepsilon_{i,m}^{q}\|^{2}+\frac{1}{2}\gamma_{i,m}^{q2}\|e_{i,m}\|^{2}-\frac{1}{\sigma_{i,m}}\tilde{\boldsymbol{\theta}}_{i,m}^{\mathrm{T}}(\sigma_{i,m}\boldsymbol{\varphi}_{i,m}s_{i,m}-\rho_{i,m}\boldsymbol{\theta}_{i,m})-$$
$$\left.\frac{w_{i,m}^{2}}{\lambda_{i,m}}+\frac{1}{2}w_{i,m}^{2}+\frac{1}{2}M_{i,m}^{2}\right] \tag{3-74}$$

根据式(3-23)、式(3-44)和式(3-45)可得

$$\dot{V}_{m-1}\leqslant-q_{m-1}\|\boldsymbol{e}\|^{2}+\eta_{m-1}+\sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}-$$
$$\frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_{1})}{\partial \boldsymbol{x}_{1}}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_{1})}{\partial \boldsymbol{x}_{1}}\right)+$$
$$\sum_{i=1}^{N}\left[\sum_{l=1}^{m-1}\frac{\rho_{i,l}}{\sigma_{i,l}}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\boldsymbol{\theta}_{i,l}-\sum_{l=2}^{m-1}c_{i,l}s_{i,l}^{2}+\sum_{l=2}^{m-1}\left(\frac{1}{\lambda_{i,l}}-1\right)w_{i,l}^{2}+\right.$$
$$\left.\frac{1}{2}\sum_{l=2}^{m-1}M_{i,m-1}^{2}+\frac{1}{2}(s_{i,m}^{2}+w_{i,m}^{2})\right] \tag{3-75}$$

将式(3-75)代入式(3-74)可得

$$\dot{V}_{m}\leqslant-q_{m}\|\boldsymbol{e}\|^{2}+\eta_{m}+\sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}-$$
$$\frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_{1})}{\partial \boldsymbol{x}_{1}}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_{1})}{\partial \boldsymbol{x}_{1}}\right)+$$
$$\sum_{i=1}^{N}\left[\sum_{l=1}^{m}\frac{\rho_{i,l}}{\sigma_{i,l}}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\boldsymbol{\theta}_{i,l}-\sum_{l=2}^{m}c_{i,l}s_{i,l}^{2}-\sum_{l=2}^{m}\left(\frac{1}{\lambda_{i,l}}-1\right)w_{i,l}^{2}+\right.$$
$$\left.\frac{1}{2}\sum_{l=2}^{m}M_{i,m}^{2}+\frac{1}{2}(s_{i,m+1}^{2}+w_{i,m+1}^{2})\right] \tag{3-76}$$

式中

$$q_{m}=q_{m-1}-\frac{1}{2}\sum_{i=1}^{N}(k_{i,m}^{2}+\gamma_{i,m}^{q2}) \tag{3-77}$$

$$\eta_{m}=\eta_{m-1}+\frac{1}{2}\sum_{i=1}^{N}\|\varepsilon_{i,m}^{q}\|^{2} \tag{3-78}$$

第 $n$ 步　定义第 $n$ 步误差变量 $s_{i,n}=\hat{x}_{i,n}-v_{i,n}$ 和滤波器误差 $w_{i,n}=v_{i,n}-x_{i,n}^{*}$。对误差变量 $s_{i,n}$ 求导可得

$$\begin{aligned}\dot{s}_{i,n} &= \dot{\hat{x}}_{i,n} - \dot{v}_{i,n} \\ &= u_i + k_{i,n}e_{i,1} + \boldsymbol{\theta}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n} + \tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n} + \varepsilon_{i,n}^{q} + \Delta g_{i,n}^{q} - \dot{v}_{i,n}\end{aligned} \tag{3-79}$$

使用滤波器技术可以获得状态变量 $v_{i,n}$ 为

$$\lambda_{i,n}\dot{v}_{i,n} + v_{i,n} = x_{i,n}^{*},\quad v_{i,n}(0) = x_{i,n}^{*}(0) \tag{3-80}$$

进一步,由方程(3-26)可得

$$\dot{w}_{i,n} = \dot{v}_{i,n} - \dot{x}_{i,n}^{*} = -\frac{w_{i,n}}{\lambda_{i,n}} + B_{i,n} \tag{3-81}$$

式中:$\lambda_{i,n}$ 为设计参数;$B_{i,n} = -\dot{x}_{i,n}^{*}$,存在一个正整数 $M_{i,n}$,使得$|B_{i,n}| \leqslant M_{i,n}$。

构造 Lyapunov 函数:

$$V_n = V_{n-1} + \sum_{i=1}^{N} V_{i,n} = V_{n-1} + \frac{1}{2}\sum_{i=1}^{N}\left\{s_{i,n}^2 + \frac{1}{\sigma_{i,n}}\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,n} + w_{i,n}^2\right\} \tag{3-82}$$

式中:$\sigma_{i,n}$ 为设计参数。

对 Lyapunov 函数求导可得

$$\dot{V}_n = \dot{V}_{n-1} + \sum_{i=1}^{N}\left\{s_{i,n}\dot{s}_{i,n} + \frac{1}{\sigma_{i,n}}\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\theta}}}_{i,n} + w_{i,n}\dot{w}_{i,n}\right\} \tag{3-83}$$

将式(3-79)代入式(3-83)可得

$$\begin{aligned}\dot{V}_n = \dot{V}_{n-1} + \sum_{i=1}^{N}\Big[ & s_{i,n}(u_i + k_{i,m}e_{i,1} + \boldsymbol{\theta}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n} + \tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n} + \varepsilon_{i,n}^{q} + \Delta g_{i,n}^{q} - \\ & \dot{v}_{i,n}) + \frac{1}{\sigma_{i,n}}\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\theta}}}_{i,n} + w_{i,n}\dot{w}_{i,n}\Big]\end{aligned} \tag{3-84}$$

根据 Young's 不等式可得

$$s_{i,n}k_{i,n}e_{i,1} \leqslant \frac{1}{2}s_{i,n}^2 + \frac{1}{2}k_{i,n}^2\|e_{i,1}\|^2 \tag{3-85}$$

$$s_{i,n}\varepsilon_{i,n}^{q} \leqslant \frac{1}{2}s_{i,n}^2 + \frac{1}{2}\|\varepsilon_{i,n}^{q}\|^2 \tag{3-86}$$

$$s_{i,n}\Delta g_{i,n}^{q} \leqslant \frac{1}{2}s_{i,n}^2 + \frac{1}{2}\gamma_{i,n}^{q2}\|e_{i,n}\|^2 \tag{3-87}$$

根据式(3-85)～式(3-87),式(3-84)可以改写为

$$\begin{aligned}\dot{V}_n \leqslant \dot{V}_{n-1} + \sum_{i=1}^{N}\Big[ & s_{i,n}(u_i + \boldsymbol{\theta}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n} + \tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n} - \dot{v}_{i,n}) + \frac{3}{2}s_{i,n}^2 + \frac{1}{2}k_{i,n}^2\|e_{i,1}\|^2 + \\ & \frac{1}{2}\|\varepsilon_{i,n}^{q}\|^2 + \frac{1}{2}\gamma_{i,n}^{q2}\|e_{i,n}\|^2 - \frac{1}{\sigma_{i,n}}\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\dot{\boldsymbol{\theta}}_{i,n} + w_{i,n}\dot{w}_{i,n}\Big]\end{aligned} \tag{3-88}$$

设计控制输入 $u_i(t)$ 和第 $n$ 步自适应律 $\boldsymbol{\theta}_{i,n}$

$$u_i=-c_{i,n}s_{i,n}-2s_{i,n}-\boldsymbol{\theta}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n}(\hat{\boldsymbol{X}}_{i,n})+\frac{x_{i,n}^{*}-v_{i,n}}{\lambda_{i,n}} \tag{3-89}$$

$$\dot{\boldsymbol{\theta}}_{i,n}=\sigma_{i,n}\boldsymbol{\varphi}_{i,n}(\hat{\boldsymbol{X}}_{i,n})s_{i,n}-\rho_{i,n}\boldsymbol{\theta}_{i,n} \tag{3-90}$$

式中：$\rho_{i,n}$ 为设计参数。

根据式(3-88)，将式(3-81)、式(3-89)和式(3-90)代入式(3-88)可得

$$\begin{aligned}\dot{V}_n\leqslant&-q_{n-1}\|\boldsymbol{e}\|^2+\eta_{n-1}+\sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}-\\&\frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)+\sum_{i=1}^{N}\left[\sum_{l=1}^{n-1}\frac{\rho_{i,l}}{\sigma_{i,l}}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\boldsymbol{\theta}_{i,l}-\right.\\&\left.\sum_{l=2}^{n-1}c_{i,l}s_{i,l}^2+\sum_{l=2}^{n-1}\left(\frac{1}{\lambda_{i,l}}-1\right)w_{i,l}^2+\frac{1}{2}\sum_{l=2}^{n-1}M_{i,l}^2+\frac{1}{2}(s_{i,n}^2+w_{i,n}^2)\right]+\\&\sum_{i=1}^{N}\left[s_{i,n}\left(-c_{i,n}s_{i,n}-2s_{i,n}-\boldsymbol{\theta}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n}(\hat{\boldsymbol{X}}_{i,n})+\frac{x_{i,n}^{*}-v_{i,n}}{\lambda_{i,n}}+\boldsymbol{\theta}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n}+\right.\right.\\&\left.\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\boldsymbol{\varphi}_{i,n}-\dot{v}_{i,n}\right)+\frac{3}{2}s_{i,n}^2+\frac{1}{2}k_{i,n}^2\|e_{i,1}\|^2+\frac{1}{2}\|\varepsilon_{i,n}^{q}\|^2+\frac{1}{2}\gamma_{i,n}^{q2}\|e_{i,n}\|^2-\\&\left.\frac{1}{\sigma_{i,n}}\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}(\sigma_{i,n}\boldsymbol{\varphi}_{i,n}(\hat{\boldsymbol{X}}_{i,n})s_{i,n}-\rho_{i,n}\boldsymbol{\theta}_{i,n})+w_{i,n}\left(-\frac{w_{i,n}}{\lambda_{i,n}}+B_{i,n}\right)\right]\end{aligned} \tag{3-91}$$

根据 Young's 不等式，$w_{i,n}B_{i,n}\leqslant\frac{1}{2}w_{i,n}^2+\frac{1}{2}M_{i,n}^2$ 成立，由此可得

$$\begin{aligned}\dot{V}_n\leqslant&-q_n\|\boldsymbol{e}\|^2+\eta_n+\sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l}-\frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial\boldsymbol{x}_1}\right)+\\&\sum_{i=1}^{N}\left[\sum_{l=1}^{n}\frac{\rho_{i,l}}{\sigma_{i,l}}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\boldsymbol{\theta}_{i,l}-\sum_{l=2}^{n}c_{i,l}s_{i,l}^2-\sum_{l=2}^{n}\left(\frac{1}{\lambda_{i,l}}-1\right)w_{i,l}^2+\frac{1}{2}\sum_{l=2}^{n}M_{i,l}^2\right]\end{aligned} \tag{3-92}$$

式中

$$q_n=q_{n-1}-\frac{1}{2}\sum_{i=1}^{N}(k_{i,n}^2+\gamma_{i,n}^{q2}) \tag{3-93}$$

$$\eta_n=\eta_{n-1}+\frac{1}{2}\sum_{i=1}^{N}\|\varepsilon_{i,n}^{q}\|^2 \tag{3-94}$$

根据 Young's 不等式可得

$$\tilde{\boldsymbol{\theta}}_{*,l}^{\mathrm{T}}\boldsymbol{\theta}_{*,l} \leqslant -\frac{1}{2}\tilde{\boldsymbol{\theta}}_{*,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{*,l} + \frac{1}{2}\boldsymbol{\theta}_{*,l}^{*\mathrm{T}}\boldsymbol{\theta}_{*,l}^{*} \tag{3-95}$$

由此可得

$$\begin{aligned}\dot{V}_n \leqslant & -q_n \|\boldsymbol{e}\|^2 + \eta_n + \sum_{i=1}^{N}\sum_{l=1}^{n}\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l} - \\ & \frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}\right) + \\ & \sum_{i=1}^{N}\left[-\sum_{l=1}^{n}\frac{\rho_{i,l}}{2\sigma_{i,l}}\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l} + \sum_{l=1}^{n}\frac{\rho_{i,l}}{2\sigma_{i,l}}\boldsymbol{\theta}_{i,l}^{*\mathrm{T}}\boldsymbol{\theta}_{i,l}^{*} - \right. \\ & \left.\sum_{l=2}^{n}c_{i,l}s_{i,l}^2 - \sum_{l=2}^{n}\left(\frac{1}{\lambda_{i,l}} - 1\right)w_{i,l}^2 + \frac{1}{2}\sum_{l=2}^{n}M_{i,l}^2\right]\end{aligned} \tag{3-96}$$

定义变量

$$\zeta = \eta_n + \sum_{i=1}^{N}\sum_{l=1}^{n}\frac{\rho_{i,l}}{2\sigma_{i,l}}\boldsymbol{\theta}_{i,l}^{*\mathrm{T}}\boldsymbol{\theta}_{i,l}^{*} + 2u(n-1) \tag{3-97}$$

将式(3-97)代入式(3-96)可得

$$\begin{aligned}\dot{V}_n \leqslant & -q_n\|\boldsymbol{e}\|^2 + \sum_{i=1}^{N}\left[-\sum_{l=2}^{n}c_{i,l}s_{i,l}^2 - \sum_{l=1}^{n}\left(\frac{\rho_{i,l}}{2\sigma_{i,l}} - \frac{1}{2}\right)\tilde{\boldsymbol{\theta}}_{i,l}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,l} - \sum_{l=2}^{n}\left(\frac{1}{\lambda_{i,l}} - 1\right)w_{i,l}^2\right] - \\ & \frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}\right)^{\mathrm{T}}\boldsymbol{H}^{\kappa(t)-1}\left(\frac{\partial P(\boldsymbol{x}_1)}{\partial \boldsymbol{x}_1}\right) + \zeta\end{aligned} \tag{3-98}$$

式中：$c_{i,l}>0(l=2,3,\cdots,n)$；$\frac{\rho_{i,l}}{2\sigma_{i,l}} - \frac{1}{2}>0(l=1,2,\cdots,n)$；$\frac{1}{\lambda_{i,l}} - 1>0(l=2,3,\cdots,n)$；$\frac{1}{2\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}>0$。

定义

$$C = \min\left\{2\frac{q_n}{\lambda_{\min}(\boldsymbol{P})}, \frac{1}{\lambda_{\max}(\boldsymbol{H}^{\kappa(t)-1})}, 2c_{i,l}, 2\left(\frac{\rho_{i,l}}{2\sigma_{i,l}} - \frac{1}{2}\right), 2\left(\frac{1}{\lambda_{i,l}} - 1\right)\right\} \tag{3-99}$$

进一步，式(3-98)可写为

$$\dot{V}_n \leqslant -CV_n + \zeta \tag{3-100}$$

根据引理 1.4 可以得出系统式(3-1)中的所有信号在闭环系统中可以保持半全局最终一致有界，并且最终收敛到分布式优化最优解 $x^*$ 的邻域内。

## 3.3　仿真实例

**仿真实例**：首先选取 Duffing-Holmes 混沌系统为例，研究其分布式优化问题。系统表示如下：

$$\begin{cases}\dot{x}_{i,1}=x_{i,2}+g_{i,1}^{\chi_i(t)}(\boldsymbol{X}_{i,1})\\ \dot{x}_{i,2}=u_i+g_{i,2}^{\chi_i(t)}(\boldsymbol{X}_{i,2})\\ y_i=x_{i,1}\end{cases},\quad i=1,2,3,4,5 \tag{3-101}$$

系统内未知非线性函数为

$$\begin{cases}g_{1,1}^{q}=g_{2,1}^{q}=g_{3,1}^{q}=g_{4,1}^{q}=g_{5,1}^{q}=0\\ g_{1,2}^{1}=x_{1,1}-0.25x_{1,2}-x_{1,1}^{3}+0.3\cos t\\ g_{1,2}^{2}=2x_{1,1}-0.25x_{1,2}-x_{1,1}^{3}\\ g_{2,2}^{1}=x_{2,1}-0.25x_{2,2}-x_{2,1}^{3}+0.1(x_{2,1}^{2}+x_{2,2}^{2})^{1/2}+0.3\cos t\\ g_{2,2}^{2}=x_{2,1}^{2}\\ g_{3,2}^{1}=x_{3,1}-0.25x_{3,2}-x_{3,1}^{3}+0.2\sin t(x_{3,1}^{2}+2x_{3,2}^{2})^{1/2}+0.3\cos t\\ g_{3,2}^{2}=x_{3,1}^{2}-x_{3,2}^{3}\\ g_{4,2}^{1}=x_{4,1}^{2}\\ g_{4,2}^{2}=x_{4,1}-0.25x_{4,2}-x_{4,1}^{3}+0.2\sin t(2x_{4,1}^{2}+2x_{4,2}^{2})^{1/2}+0.3\cos t\\ g_{5,2}^{1}=x_{5,1}^{3}+x_{5,2}^{2}\\ g_{5,2}^{2}=x_{5,1}-0.1x_{5,2}-x_{5,1}^{3}+0.2\sin t(x_{5,1}^{2}+x_{5,2}^{2})^{1/2}+0.3\cos t\end{cases} \tag{3-102}$$

系统初始状态 $\boldsymbol{x}_i$ 设置为

$$\boldsymbol{x}_1(0)=[0.1,0.1]^{\mathrm{T}},\quad \boldsymbol{x}_2(0)=[0.2,0.2]^{\mathrm{T}},\quad \boldsymbol{x}_3(0)=[0.3,0.3]^{\mathrm{T}},$$

$$\boldsymbol{x}_4(0)=[0.4,0.4]^{\mathrm{T}},\quad \boldsymbol{x}_5(0)=[0.5,0.5]^{\mathrm{T}}$$

观测器参数设置为

$$k_{1,1}=k_{2,1}=k_{3,1}=k_{4,1}=k_{5,1}=500,\quad k_{1,2}=k_{2,2}=k_{3,2}=k_{4,2}=k_{5,2}=5000$$

初始状态 $\hat{\boldsymbol{x}}_i$ 设置为

$$\hat{\boldsymbol{x}}_1=[0.2,0.2]^{\mathrm{T}},\quad \hat{\boldsymbol{x}}_2=[0.3,0.3]^{\mathrm{T}},\quad \hat{\boldsymbol{x}}_3=[0.4,0.4]^{\mathrm{T}},$$

$$\hat{\boldsymbol{x}}_4=[0.5,0.5]^{\mathrm{T}},\quad \hat{\boldsymbol{x}}_5=[0.6,0.6]^{\mathrm{T}}$$

定义参考信号 $x_{\mathrm{d}}=\sin t$。给定局部目标函数如下：

$$\begin{cases} f_1(x_{1,1})=3.2x_{1,1}^2-6.4x_{\mathrm{d}}x_{1,1}+3.2x_{\mathrm{d}}^2+1 \\ f_2(x_{2,1})=4.6x_{2,1}^2-9.2x_{\mathrm{d}}x_{2,1}+4.6x_{\mathrm{d}}^2+2 \\ f_3(x_{3,1})=2.5x_{3,1}^2-5x_{\mathrm{d}}x_{3,1}+2.5x_{\mathrm{d}}^2+3 \\ f_4(x_{4,1})=2.8x_{4,1}^2-5.6x_{\mathrm{d}}x_{4,1}+2.8x_{\mathrm{d}}^2+4 \\ f_5(x_{5,1})=3.5x_{5,1}^2-7x_{\mathrm{d}}x_{5,1}+3.5x_{\mathrm{d}}^2+5 \end{cases} \tag{3-103}$$

根据式(3-42)、式(3-43)、式(3-89)和式(3-90)设计虚拟控制律、自适应律和控制输入。选择设计参数为 $c_{i,1}=4, c_{i,2}=3, \sigma_{i,1}=\sigma_{i,2}=1, \rho_{i,1}=\rho_{i,2}=80, \lambda_{i,2}=0.1$。

图 3-1～图 3-8 为仿真结果。图 3-1 为多智能体通信拓扑图。图 3-2 和图 3-3 为通过本章所提方法得到的系统状态跟踪图像以及智能体 1 的观测器估计值。图 3-4 为智能体输出

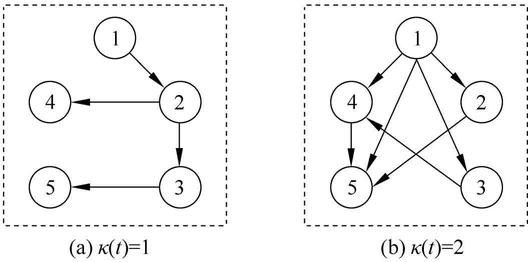



图 3-1 多智能体通信拓扑图

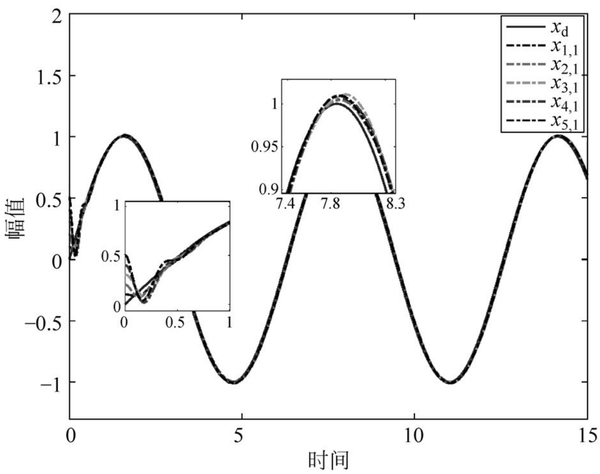
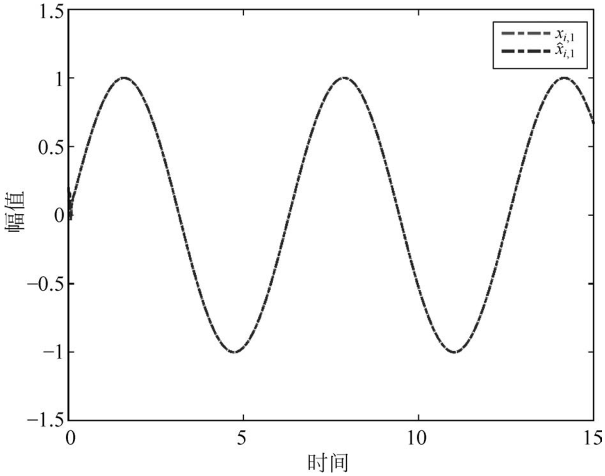



图 3-2 状态 $x_{i,1}$ 以及跟踪信号轨迹 $y_{\mathrm{d}}$

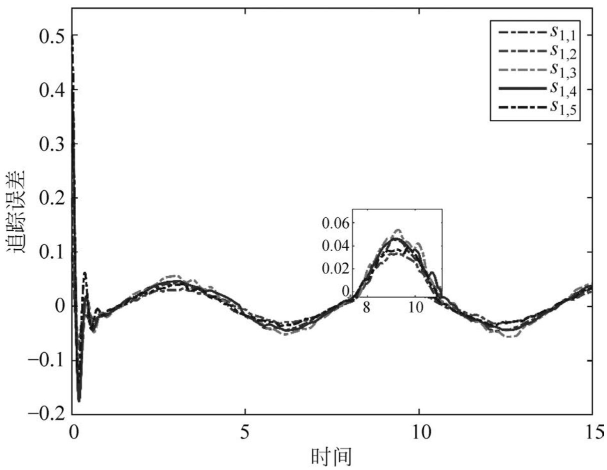
彩图

信号与参考信号之间的追踪误差，可以看出追踪误差在一个合理的范围内。图 3-5 为本章所提出方法的得到的控制输入轨迹。图 3-6 为本章所构造的惩罚函数的数值轨迹，可以看出惩罚函数最终能够收敛到最小值附近。图 3-7 和图 3-8 给出拓扑切换信号和系统切换信号。




彩图

图 3-3 状态 $x_{1,1}$ 以及观测器信号 $\hat{x}_{1,1}$




彩图

图 3-4 追踪误差 $s_{i,1}$

彩图

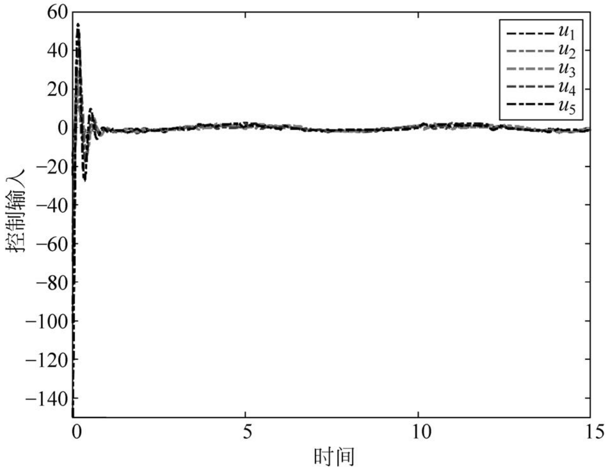



图 3-5　控制输入 $u_i$

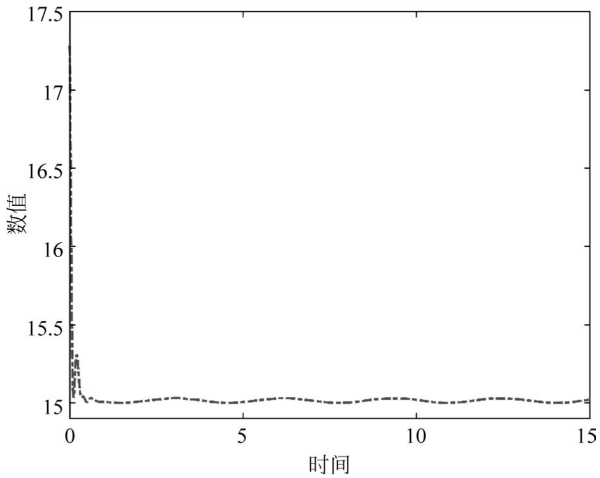



图 3-6　惩罚函数 $P(\boldsymbol{x}_1)$

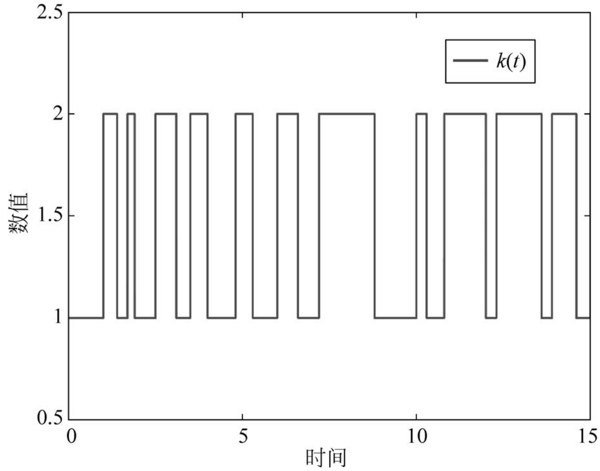
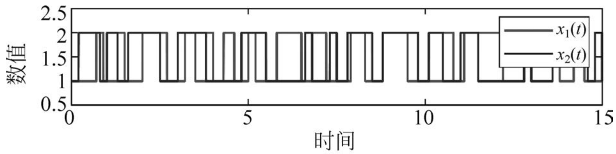
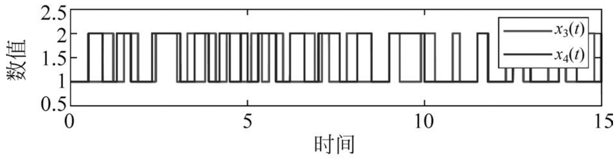


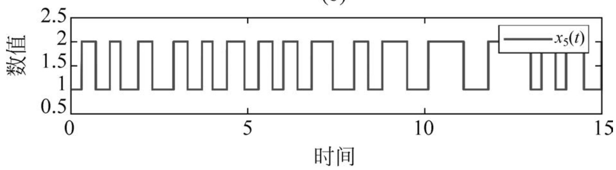

图 3-7　拓扑切换信号




(a)




(b)




(c)

图 3-8　系统切换信号

彩图